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A model for performance prediction of hydrocyclones

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Abstract

The equilibrium orbit theory and the residence time theory predict that the product between the Stokes number and the Euler number ($Stk_{50}Eu$) should be constant for a family of geometrically similar hydrocyclones. It has been already shown that $Stk_{50}Eu$ is, in reality, a function of feed concentration and water flow ratio. For different families, this product is also a function of the cyclone geometrical proportions. In this work, data obtained with seven hydrocyclones was used to generate a model based on dimensionless groups capable to predict performance of hydrocyclones. Unlike other models that can be found in the literature, where the parameters have to be adjusted for each data set, the proposed model was able to reproduce data from the classical works of Rietema, Bradley and Kelsall. The assumption of invariance of the reduced grade efficiency curve adopted in this work seems to be a good approximation for performance prediction of hydrocyclones. The fish-hook effect could not be found in any of the 160 experiments, probably due to the apparatus used to determine size distributions, which measure a dynamically equivalent diameter known as Stokes diameter.

Keywords: Hydrocyclones; Model; Performance; Geometrical proportions; Efficiency

1. Introduction

Two well-known theories for particle separation in hydrocyclones are the equilibrium orbit theory [1] and the residence time theory [2]. The equilibrium orbit theory assumes that particles of a given size will reach an equilibrium radial orbit position inside the hydrocyclone where their outward terminal settling velocity is equal to the inward radial velocity of the liquid. Accordingly to this theory, larger particles will attain a radial orbit position near the wall, where the axial fluid velocity has a downward direction. These particles will, therefore, leave the cyclone through the underflow. The radial orbit position of smaller particles will be located near the centre, inside the region where the axial fluid velocity is upward. These particles will, therefore, escape through the overflow. The cut size is defined as the particle size whose equilibrium orbit is coincident with the locus of zero vertical velocity of the fluid. Such a particle will have equal chance to escape the hydrocyclone either through the underflow or through the overflow. According to the residence time theory, a particle will be separated as a function of both the position it enters the cyclone and the available residence time. The cut size will be the size of the particle which entering the equipment exactly in the centre of the inlet pipe will just reach the wall in the residence time available.

Although the first patent of a hydrocyclone is about 110 years old, research works are still in progress aiming at developing new applications [3–7] or at understanding the complex flow inside this equipment. In the recent years, the advance of computer power has allowed numerical solutions for the differential equations that constitute the equation of motion. The use of computational fluid dynamics (CFD) is beginning to give a better understanding of the strongly swirling turbulent flow inside hydrocyclones. These high swirl effects induce anisotropic turbulence. As the conventional k–ε model assumes isotropic turbulence, it usually gives incorrect predictions of the flow patterns. This problem can be overcome either by modifying the original k–ε model [8,9] or by using the Reynolds stress model [10–12]. The problem still to be properly solved is the coupling between the particulate phase and the liquid. Turbulent eddies,

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Nomenclature

C_v	feed volumetric concentration
C_{vu}	underflow volumetric concentration
d	particle diameter (m)
d'_{50}	reduced cut size (m)
D_c	hydrocyclone diameter (m)
D_i	feed inlet diameter (m)
D_o	overflow diameter (m)
D_u	underflow diameter (m)
E_T	total efficiency
E'_T	reduced total efficiency
Eu	Euler number
G'	reduced grade efficiency
k	parameter in Eqs. (1)–(3)
ℓ	vortex finder length (m)
L	hydrocyclone length (m)
L_1	height of hydrocyclone cylindrical part (m)
m	parameter in Eq. (9)
n	parameter in Eqs. (1)–(3)
ΔP	pressure drop (Pa)
Q	feed volumetric flow rate ($m^3\,s^{-1}$)
Q_u	underflow volumetric flow rate ($m^3\,s^{-1}$)
R_w	water flow ratio
Re	Reynolds number
Stk_{50}	Stokes number
y	cumulative particle size distribution (undersize) of feed suspension

Greek symbols

α	parameter in Eq. (8)
μ	liquid viscosity (Pa s)
θ	angle of the hydrocyclone cone
ρ	liquid density ($kg\,m^{-3}$)
ρ_s	solid density ($kg\,m^{-3}$)

which are random in nature, and hindered settling make a theoretical solution for performance very complex. Therefore, there is still a need for empirical and semi-empirical models to describe hydrocyclone performance.

Several authors have used either the equilibrium orbit theory or the residence time theory to derive different equations for the cut size. It has been shown that most of these equations lead to the conclusion that the product between Stokes number and Euler number ($Stk_{50}Eu$) is constant for geometrically similar hydrocyclones [13,14]. It has also been shown that this product depends on the cyclone design, but is not affected by the relative size of the inlet orifice (D_i/D_c) [15]. Both theories were developed under the assumption that hindered settling does not occur, i.e., that the feed suspension is diluted. As concentration reduces the terminal settling velocities of the particles, it can be expected that the product $Stk_{50}Eu$ will vary with feed concentration. Some hydrocyclones have a set of different underflow orifice sizes (D_u) in order to permit adjustments in their performance. It is also expected that the product $Stk_{50}Eu$ will be a function of D_u or of an operational variable greatly affected by D_u, like the water flow ratio.

Experimental work has shown that the product $Stk_{50}Eu$ is a function of water flow ratio R_w and volumetric feed concentration C_v [14,16]. Based on this work, a model composed of Eqs. (1)–(3), which describes the operation of geometrically similar hydrocyclones, was proposed.

$$Stk_{50}Eu = k_1 \left[\ln\left(\frac{1}{R_w}\right)\right]^{n_1} \exp(n_2 C_v) \tag{1}$$

$$Eu = k_2\, Re^{n_3} \exp(n_4 C_v) \tag{2}$$

$$R_w = k_3 \left(\frac{D_u}{D_c}\right)^{n_5} Eu^{n_6} \tag{3}$$

where k and n are the parameters of the equations and the product $Stk_{50}Eu$, the Euler number Eu, the Reynolds number Re, and the water flow ratio R_w are given by Eqs. (4)–(7), respectively.

$$Stk_{50}Eu = \frac{\pi(\rho_s - \rho)\Delta P D_c (d'_{50})^2}{36\mu\rho Q} \tag{4}$$

$$Eu = \frac{\pi^2 \Delta P D_c^4}{8\rho Q^2} \tag{5}$$

$$Re = \frac{4\rho Q}{\pi\mu D_c} \tag{6}$$

$$R_w = \frac{Q_u(1 - C_{vu})}{Q(1 - C_v)} \tag{7}$$

Table 1 gives the geometrical proportions of the Rietema [2], Bradley [17], and Demco 4H hydrocyclones. The first two are families of geometrically similar hydrocyclones and the last is a commercial one produced by Demco. Table 2 gives the values for the constants in Eqs. (1)–(3) for these three types of cyclones [14,16,18].

If the feed size distribution y and the reduced grade efficiency curve G' are known, Eqs. (1)–(3) can fully describe the performance of a hydrocyclone [19]. G' can be given by the Lynch and Rao [20] model or by a modification of the classical Rosin and Rammler equation [21,44] as proposed by Plitt [22]. These two models are given by Eqs. (8) and (9), respectively.

$$G' = \frac{\exp(\alpha d/d'_{50}) - 1}{\exp(\alpha d/d'_{50}) + \exp(\alpha) - 2} \tag{8}$$

$$G' = 1 - \exp\left[-0.693\left(\frac{d}{d'_{50}}\right)^m\right] \tag{9}$$

Table 1
Geometrical proportions of the hydrocyclones used in this work

Hydrocyclone	D_i/D_c	D_o/D_c	L/D_c	L_1/D_c	ℓ/D_c	θ
Rietema	0.28	0.34	5.0	–	0.40	20°
Bradley	1/7	1/5	–	1/2	1/3	9°
Demco 4H	0.26	0.33	3.3	0.55	0.55	18°

Table 2
Parameters of Eqs. (1)–(3) for Rietema, Bradley, and Demco 4H hydrocyclones [14,16,18]

Constant	Hydrocyclone		
	Rietema	Bradley	Demco 4H
k_1	0.0474	0.0550	0.0088
k_2	371.5	258	3300
k_3	1218	1.21×10^6	0.127
n_1	0.74	0.66	2.31
n_2	9.0	12.0	15.5
n_3	0.12	0.37	0.00
n_4	−2.12	0.00	0.00
n_5	4.75	2.63	0.78
n_6	−0.30	−1.12	0.00

where α and m are the parameters of the equations. Table 3 gives the values of these parameters for Rietema, Bradley, and Demco 4H hydrocyclones [14,16,18].

The reduced total efficiency E'_T and the total efficiency E_T can be calculated based on Eqs. (10) and (11).

$$E'_\mathrm{T} = \int_0^1 G' \,\mathrm{d}y \tag{10}$$

$$E'_\mathrm{T} = \frac{E_\mathrm{T} - R_\mathrm{w}}{1 - R_\mathrm{w}} \tag{11}$$

The model described by Eqs. (1)–(3) can be used either in design or in performance prediction of hydrocyclones of a given geometry. The problem posed here is that hydrocyclones with geometrical proportions other than those mentioned in Table 1 need to be tested in order to find the constants of the aforementioned equations.

Some authors [20,22–27] have proposed empirical models to describe the performance of hydrocyclones of any geometrical proportions. Unfortunately, these empirical equations are only capable to fit well with their original data, i.e., the equation constants must be recalculated for each new data set [25–28].

A model that could be applicable to a cyclone of any design would have more chances to succeed if it has a theoretical background. In the present work, a semi-empirical model based on dimensionless groups like the model given by Eqs. (1)–(3) is proposed. In order to validate it, the values calculated with this model are compared with data sets that can be found in the classical works of Rietema [2], Bradley and Pulling [29,45], and Kelsall [30,46].

Table 3
Parameters of Eqs. (8) and (9) for the reduced grade efficiency curve [14,16,18]

Hydrocyclone	α	m
Rietema	4.23	2.45
Bradley	5.1	3.12
Demco 4H	5.4	3.30

Table 4
Size of the hydrocyclones and underflow orifices used in this work

Hydrocyclone	D_c (mm)	D_u (mm)
Rietema 1	22	2.0–4.6–6.0
Rietema 2	44	4.0–8.2–11.5
Rietema 3	88	8.5–16.0–24.7
Bradley 1	15	1.0–1.5–2.0
Bradley 2	30	2.0–3.0–4.0
Bradley 3	60	4.0–6.0–8.0
Demco 4H	122	4.5–6.0–11.0–19.0

2. Materials and methods

Data obtained with seven hydrocyclones were used in this work. Three of the cyclones obey the geometry recommended by Rietema [2], three obey Bradley's geometry [17], and the last was a Demco 4H. Two Bradley and all Rietema hydrocyclones were manufactured from brass, the largest Bradley cyclone was manufactured from PVC, and the Demco 4H is made of steel with an internal protection of polyurethane. Table 4 gives the diameters of these hydrocyclones and also their underflow orifice sizes. The range of geometrical proportions covered in this work is given in Table 5. This range was thought to be wide enough aiming to include most of the well-known hydrocyclone designs.

The test rigs obeyed all recommendations given by Svarovsky [13] and Heiskanen [31] and details can be found elsewhere [14–16,18]. Both the underflow and the overflow were discharged into atmospheric pressure. The pressure drop used ranged from 70 to 280 kPa, and the pressure gauge was placed close to the hydrocyclone inlet in order to exclude the pressure losses in the inlet pipe.

The suspending liquid both in the hydrocyclone experiments and in the particle size analysis was water with $1.0\,\mathrm{g\,l^{-1}}$ of Calgon as a dispersing agent. The test materials used were $CaCO_3$ ($\rho_\mathrm{s} = 2450\,\mathrm{kg\,m^{-3}}$), chalk ($\rho_\mathrm{s} = 2780\,\mathrm{kg\,m^{-3}}$), alumina hydrate ($\rho_\mathrm{s} = 2420\,\mathrm{kg\,m^{-3}}$), and barite ($\rho_\mathrm{s} = 3750\,\mathrm{kg\,m^{-3}}$). The particle size distributions of the feed solids are shown in Fig. 1. The feed volumetric concentration varied from 0 to 10%. The dispersion before each particle size analysis was aided by using an ultrasonic bath at 80 W for 1 min [32]. The particle size analyses were carried out using the Andreasen pipette method [33] or a Ladeq equipment [16].

Table 5
Range of geometrical proportions of the hydrocyclones used in this work

Geometrical proportion	Range
$D_\mathrm{i}/D_\mathrm{c}$	0.14–0.28
$D_\mathrm{o}/D_\mathrm{c}$	0.20–0.34
$D_\mathrm{u}/D_\mathrm{c}$	0.04–0.28
ℓ/D_c	0.33–0.55
L/D_c	3.30–6.93
θ	9°–20°

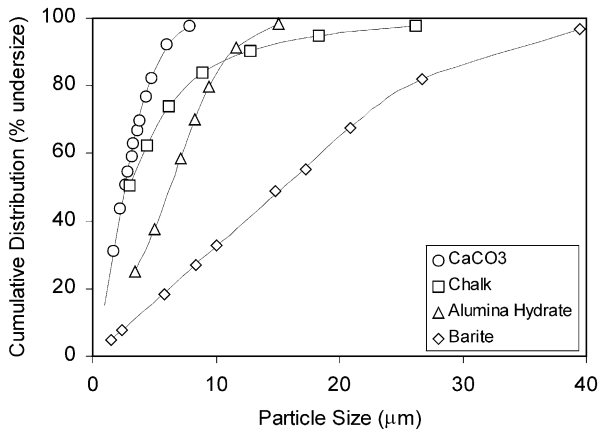

Fig. 1. Particle size distributions of the feed materials used in this work.

3. Results and discussion

An amount of 160 experiments were considered and multiple linear regression was used to derive Eqs. (12)–(14).

$$Stk_{50}Eu = 0.12\left(\frac{D_c}{D_o}\right)^{0.95}\left(\frac{D_c}{L-\ell}\right)^{1.33}\left[\ln\left(\frac{1}{R_w}\right)\right]^{0.79} \times \exp(12.0C_v) \quad (12)$$

$$Eu = 43.5D_c^{0.57}\left(\frac{D_c}{D_i}\right)^{2.61}\left(\frac{D_c}{D_o^2+D_u^2}\right)^{0.42} \times \left(\frac{D_c}{L-\ell}\right)^{0.98} Re^{0.12}\exp(-0.51C_v) \quad (13)$$

$$R_w = 1.18\left(\frac{D_c}{D_o}\right)^{5.97}\left(\frac{D_u}{D_c}\right)^{3.10} Eu^{-0.54} \quad (14)$$

Eqs. (12) and (14) are totally based on dimensionless groups, therefore, they can be used with any coherent system of units as, for instance, the international system of units (SI). This is not case for Eq. (13), where only the SI units must be employed.

Eqs. (12)–(14) can predict the performance of hydrocyclones with geometrical proportions within the limits used in this work and given in Table 5. For instance, the reduced cut size, the flow rate, and the underflow orifice size can be calculated based on Eqs. (4) and (12); (5), (6) and (13); (5) and (14), respectively, leading to Eqs. (15), (16) and (17).

$$d'_{50} = \frac{1.173D_c^{0.64}}{D_o^{0.475}(L-\ell)^{0.665}}\left[\frac{\mu\rho Q}{(\rho_s-\rho)\Delta P}\right]^{0.5} \times \left[\ln\left(\frac{1}{R_w}\right)\right]^{0.395}\exp(6.0C_v) \quad (15)$$

$$Q = 0.184D_c^{-0.217}D_i^{1.231}(D_o^2+D_u^2)^{0.198} \times (L-\ell)^{0.462}\mu^{0.0566}\rho^{-0.528}\Delta P^{0.472}\exp(0.241C_v) \quad (16)$$

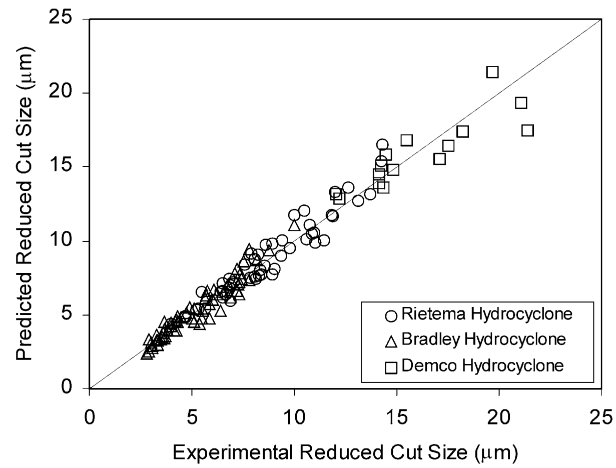

Fig. 2. Comparison between the experimental values of the reduced cut sizes obtained in this work and the values predicted by Eq. (15).

$$D_u = 0.983\frac{D_o^{1.926}}{D_o^{0.229}}\left(\frac{\Delta P}{\rho Q^2}\right)^{0.174} R_w^{0.323} \quad (17)$$

Eqs. (15) and (17) can be used with any coherent system of units as, for instance, the SI units. Eq. (16), however, must be used only in SI units. Figs. 2–4 show a comparison between the experimental results of reduced cut size, flow rate and underflow orifice diameter and the values calculated with Eqs. (15)–(17), respectively. The good agreement between the experimental and the calculated values shown in these figures means that Eqs. (15)–(17) are capable to adjust well the experimental data that has originated these equations. In order to test the capability of the model to fit different data sets, the experimental values reported in classical works [2,29,30,45,46] were compared with the values predicted by the model.

Rietema [2] presented the results obtained with a hydrocyclone of 75 mm diameter. In order to establish his best

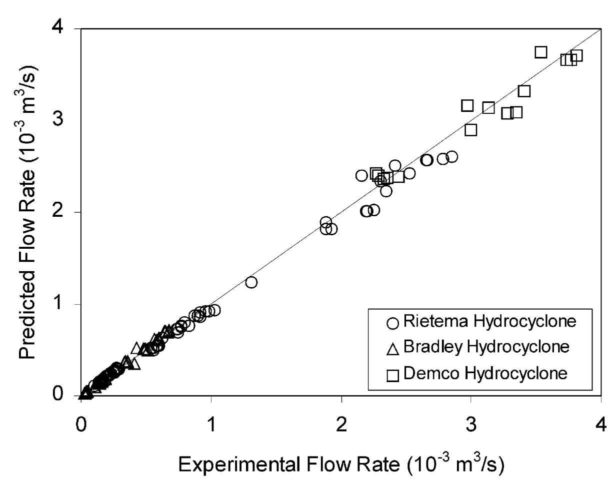

Fig. 3. Comparison between the experimental values of the flow rates obtained in this work for pressure drops between 70 and 280 kPa and the values predicted by Eq. (16).

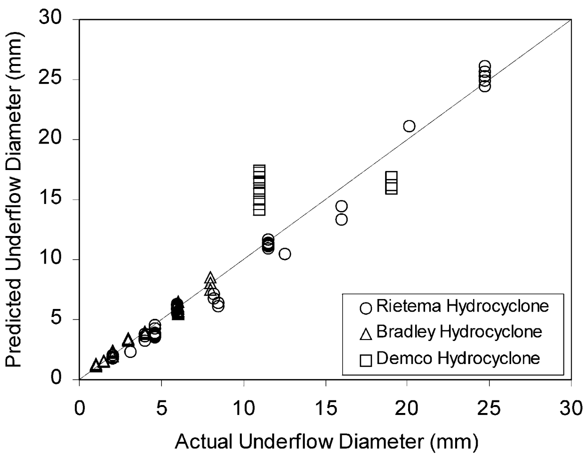

Fig. 4. Comparison between the actual values of the underflow orifice diameters used in this work at different operational conditions and the values predicted by Eq. (17).

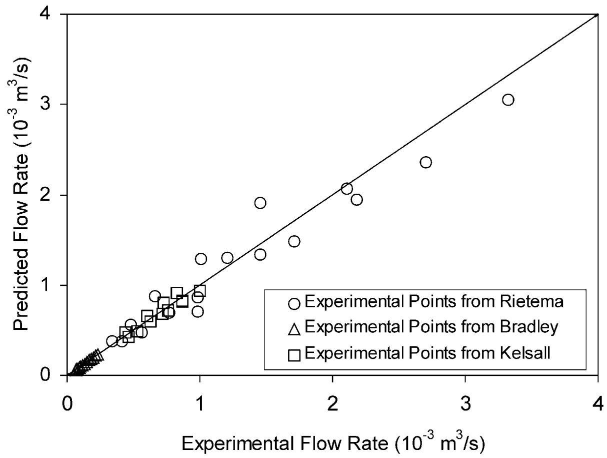

Fig. 5. Experimental values of flow rate from Rietema [2], Bradley and Pulling [29,45] and Kelsall [30,46] and the values predicted by Eq. (16).

geometrical proportions, he varied all important dimensions of the hydrocyclone. Those experiments that lie within the ranges presented in Table 5 will be used here for comparison with the predicted values using Eqs. (15)–(17). Bradley and Pulling [29,45] used a hydrocyclone of 38.1 mm diameter in their work on flow patterns in hydrocyclones. As they used a hydrocyclone that obeys Bradley's recommended proportions, all their data can be used for comparison. Kelsall [30,46] in his paper on velocity profile determination used a 76.2 mm diameter hydrocyclone. He also varied some geometrical proportions. Unfortunately, all his 12 experiments were carried out using a hydrocyclone with geometrical proportions outside the range used in the present work.

Rietema [2] gave the grade efficiency curve of each experiment listed in Table 1 of Part III of his paper. Five of these experiments lie within the range of geometrical proportions used in the present work. Bradley and Pulling [29,45] gave only one reduced cut size obtained through Fig. 23 of their work. The comparison between these six experimental reduced cut size and the ones predicted by Eq. (15) is shown in Table 6.

For flow rate comparison, it is possible to use 17 of the experiments from Rietema, including the experiments with water only, all 12 experimental points given in Fig. 21 of Bradley's work, and all 12 Kelsall's experiments. Fig. 5 shows the predicted values for flow rate given by Eq. (16) against the experimental values. The Rietema's experimental points show a reasonable agreement and the points from Bradley agree very well. Although his hydrocyclone does not have geometrical proportions lying within the range used in the present work, Kelsall's points show a good agreement, even for the four points where that author worked without overflow, i.e., with 100% water flow ratio.

A comparison between the experimental points from Rietema and Kelsall and the ones predicted by Eq. (17) for the underflow orifice size can be seen in Fig. 6. The agreement is reasonably good, even using all 25 points of Table 1 from Part III of Rietema's work. From the 12 Kelsall's experiments, only eight could be used here because the author worked without overflow in the other four experiments. Bradley's experiment could not be included in Fig. 6 because he did not inform the water flow ratio obtained.

Based on the experimental data from the present work, the values of α in Eq. (8) and m in Eq. (9) were found to be 5 and 3, respectively. Fig. 7 shows the predicted reduced grade efficiency curve given by Eq. (9) with the parameter $m = 3$ found in this work. The 13 experimental points of G' given by Rietema (related to the five experiments from

Table 6
Comparison between experimental reduced cut sizes and the ones predicted by Eq. (15)[a]

	Experiment No.	Water flow ratio (%)	Experimental reduced cut size (μm)	Predicted reduced cut size (μm)
Rietema	141	21.6	11.5	9.5
Rietema	144	19	9.3	9.2
Rietema	149	20.7	8.8	8.8
Rietema	153	10	10.0	11.6
Rietema	181	8.3	9.0	10.2
Bradley	Single	Not informed	20.0	20.8

[a] Values from Rietema [2] were taken from Table 1 of Part III of his paper (only shown here the experiments with hydrocyclones that lie within the range of geometrical proportions used in the present work). The value from Bradley and Pulling [29,45] was taken from Fig. 23 of their work.

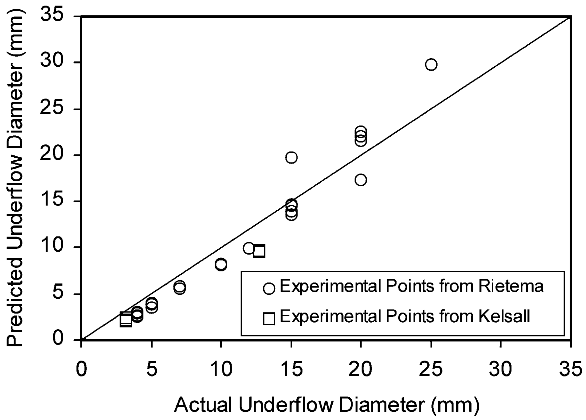

Fig. 6. Underflow diameters actually used by Rietema [2] and Kelsall [30,46] and the values predicted by Eq. (17).

Table 1, Part III, of his study) and the seven experimental points given by Bradley in his Fig. 23 are also plotted in Fig. 7. As can be seen, the agreement is reasonably good.

Although Table 3 shows different values for α and m as a function of the hydrocyclone design and, implicitly, of the material used, the authors' experience is that such variations does not have a significant influence on the predicted total efficiencies for the great majority of cases. The same conclusion was reached by Nageswararao [34], who made a very detailed analysis of Eqs. (8) and (9). He concluded that the error introduced in G' by considering a constant α or m is well within the sampling and analysis errors associated in measuring G'. Therefore, the assumption of invariance of the reduced grade efficiency curve is an excellent approximation when predicting performance of hydrocyclones (see Fig. 7).

An interesting discussion can be done regarding the ways of correcting the grade efficiency to produce the reduced grade efficiency. The grade efficiency gives the mass fraction of solids of a given diameter recovered in the underflow. For a better comprehension, the grade efficiency is the total efficiency a separator would give if fed only with particles of a given size. A flow ratio equal to R_w means that this fraction of feed fluid is leaving the separator through the underflow. Since the fluid carries solid particles with it, some particles will be discharged into the underflow not due to the centrifugal action of the separator but due to entrainment. Since originally proposed by Kelsall [35], R_w has been widely used to represent this bypass [31] and is assumed to be the minimal efficiency at which a separator will operate even if no centrifugal action takes place. Therefore, the use of R_w to correct the grade efficiency means that the proportion of all sizes reporting to the underflow through the bypass mechanism is equal to R_w. The reduced grade efficiency curves represented by Eqs. (8) and (9) are obtained through normalisation of the grade efficiency curves using R_w. They are convenient mathematical functions because G' given by these equations varies from zero, as the particle sizes approach zero, to 1, as the size reaches large values. However, these equations are not capable to simulate the fish-hook effect (a dip exhibited in the grade efficiency curve) that is sometimes reported in the literature [36,37]. Since laser particle size analyser started to be used and because these equipments can measure very fine particle sizes, this phenomenon has been more intensively reported and different models, aiming to account for it, have been proposed [38–41]. Several reasons have been given to explain the fish-hook effect. For instance, Finch [42] treated the entrainment as a function of particle size. He considers that relatively large particles will not move so readily as the water but, with decreasing size, they progressively separate in proportion to the water. Frachon and Cilliers [38] explain the fish-hook based on a methodology that quantifies the relative importance of particle dispersion due to turbulence, and particle motion due to classification forces. Ray et al. [43] recently published an interesting work where different analytical methods are used to evaluate grade efficiencies of cyclones. They found fish-hook effects when using a laser scattering sizer but not found them when using a disc centrifuge to measure the particle size of the same samples. When analysing the results, they say that "in the disc centrifuge only one particle size is registered at any given time, while in the laser scattering sizer, the size distribution is back-calculated from a signal arising from the suspension as a whole". They also stated that "the difficulties of light scattering sizing techniques in the fine end are well known" and that "since the apparatus does not determine a dynamically equivalent diameter, errors may arise from factors such as non-sphericity or a varying particle density. Moreover, the analysis turned out to be very sensitive to the optical model, and indications are that the technique may not give accurate results in the extremes of the size range". Therefore, care must be taken when analysing fish-hook effects obtained when using laser scattering sizers for particle size measurements. In the present work, two apparatus were used to determine size distributions, and both measure Stokes diameter, a dynamically equivalent diameter. This may be the reason why the fish-hook effect was not found in any of the 160 experiments.

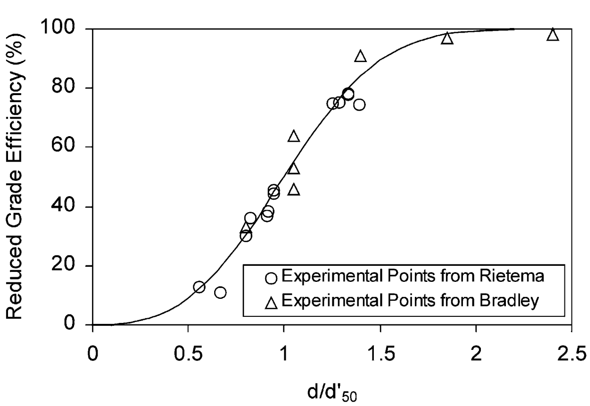

Fig. 7. Experimental values of the reduced grade efficiency from Rietema [2] and Bradley and Pulling [29,45] and the curve given by Eq. (9) using the experimental value of $m = 3$ obtained in the present work.

4. Conclusions

Based on dimensionless groups derived from the equilibrium orbit theory [1] and the residence time theory [2], a model for performance prediction of hydrocyclones has been developed. The model was able to reproduce not only the data used in this work but also data from the classical works of Rietema [2], Bradley and Pulling [29,45] and Kelsall [30,46].

The assumption of invariance of the reduced grade efficiency as a function of d/d'_{50} seems to be a good approximation when predicting performance of hydrocyclones.

An analysis was done on the ways of correcting the grade efficiency. The published data indicate that fish-hook effects obtained with laser particle size analyser should be confirmed using an apparatus that determines dynamically equivalent diameters. Further work is required on this subject.

References

[1] M.G. Driessen, Theory of flow in a cyclone, Rev. L'Industrie Min. Spl. (1951) 449–461.
[2] K. Rietema, Performance and design of hydrocyclones. Parts I–IV, Chem. Eng. Sci. 15 (1961) 298–325.
[3] V.M. Matta, R.A. Medronho, A new method for yeast recovery in batch ethanol fermentations: filter aid filtration followed by separation of yeast from filter aid using hydrocyclones, Bioseparation 9 (2000) 43–53.
[4] M. Lübberstedt, R.A. Medronho, F.B. Anspach, W.-D. Deckwer, Separation of mammalian cells using hydrocyclones, in: Proceedings of the World Congress on Biotechnology — Biotechnology 2000, Berlin, 1 (2000) 460–462.
[5] M. Lübberstedt, R.A. Medronho, F.B. Anspach, W.-D. Deckwer, Abtrennung tierischer Zellen mit Hydrozyklonen, Chem. Ing. Technol. 72 (2000) 1089–1090.
[6] J.J. Cilliers, S.T.L. Harrison, The application of mini-hydrocyclones in the concentration of yeast suspensions, Chem. Eng. J. 65 (1997) 21–26.
[7] H. Yuan, D. Rickwood, I.C. Smyth, M.T. Thew, An investigation into the possible use of hydrocyclones for the removal of yeast from beer, Bioseparation 6 (1996) 159–163.
[8] P. He, M. Salcudean, I.S. Gartshore, A numerical simulation of hydrocyclones, Chem. Eng. Res. Des. 77 (1999) 429–441.
[9] G.Q. Dai, J.M. Li, W.M. Chen, Numerical prediction of the liquid flow within a hydrocyclone, Chem. Eng. J. 74 (1999) 217–233.
[10] R.A. Medronho, J. Schuetze, W.-D. Deckwer, Numerical simulation of hydrocyclones for cell separation, Braz. J. Chem. Eng., submitted for publication.
[11] L. Chu, W. Chen, X. Li, C. Wu, Numerical simulation of turbulence and its structure in a hydrocyclone, Trans. Nonferrous Met. Soc. China 9 (1999) 128–136.
[12] J. Averous, R. Fuentes, Advances in the numerical simulation of hydrocyclone classification, Can. Metall. Quart. 36 (1997) 309–314.
[13] L. Svarovsky, Hydrocyclones, Holt, Rinehart & Winston, London, 1984.
[14] R.A. Medronho, Scale-up of hydrocyclones at low feed concentrations, Ph.D. Thesis, University of Bradford, Bradford, UK, 1984.
[15] R.A. Medronho, L. Svarovsky, Tests to verify hydrocyclone scale-up procedure, in: Proceedings of the Second International Conference on Hydrocyclones, BHRA, Bath, UK, 1984, pp. 1–14.
[16] M. Antunes, R.A. Medronho, Bradley hydrocyclones: design and performance analysis, in: L. Svarovsky, M.T. Thew (Eds.), Hydrocyclones: Analysis and Applications, Kluwer Academic Publishers, Dordrecht, 1992, pp. 3–13.
[17] D. Bradley, The Hydrocyclone, Pergamon Press, London, 1965.
[18] R. Azevedo, C.O. Veloso, R.A. Medronho, Performance de hidrociclones CBV-Demco 4H, in: Proceedings of the XXII Brazilian Congress on Particulate Systems, Friburgo, Brazil, 1990, pp. 481–488.
[19] L.R. Castilho, R.A. Medronho, A simple procedure for design and performance prediction of Bradley and Rietema hydrocyclones, Min. Eng. 13 (2000) 183–191.
[20] A.J. Lynch, T.C. Rao, Modelling and scale-up of hydrocyclone classifiers, in: Proceedings of the 11th International Mineral Processing Congress, Cagliari, Italy, 1975, pp. 1–25.
[21] P. Rosin, E. Rammler, The laws governing the fineness of powdered coal, J. Inst. Fuel 7 (1933) 29–36.
[22] L.R. Plitt, A mathematical model of the hydrocyclone classifier, CIM Bull. 69 (1976) 114–123.
[23] K. Nageswararao, A generalised model for hydrocyclone classifiers, AusIMM Proc. 300 (1995) 21.
[24] M.A. Doheim, M.N.M. Ahmed, S.A. Sayed, Hydrocyclone investigation and modelling at medium feed solids concentration of mixed minerals, Trans. Inst. Mining Metall., Sect. C 104 (1995) C37–C44.
[25] G. Vallebuona, A. Casali, P. Bevilacqua, Small-diameter hydrocyclones: performance prediction by empirical models, Min. Metall. Proc. 12 (1995) 184–188.
[26] G. Vallebuona, A. Casali, G. Ferrara, O. Leal, P. Bevilacqua, Modelling for small diameter hydrocyclones, Min. Eng. 8 (1995) 321–327.
[27] A.K. Asomah, T.J. Napier-Munn, An empirical model of hydrocyclones, incorporating angle of cyclone inclination, Min. Eng. 10 (1997) 339–347.
[28] M.A.Z. Coelho, R.A. Medronho, An evaluation of the Plitt and Lynch and Rao models for the hydrocyclones, in: L. Svarovsky, M.T. Thew (Eds.), Hydrocyclones: Analysis and Applications, Kluwer Academic Publishers, Dordrecht, 1992, pp. 63–72.
[29] D. Bradley, D.J. Pulling, Flow patterns in the hydraulic cyclone and their interpretation in terms of performance, Trans. Inst. Chem. Engrs. 37 (1959) 34–44.
[30] D.F. Kelsall, A study of the motion of solid particles in a hydraulic cyclone, Trans. Inst. Chem. Engrs. 30 (1952) 87–104.
[31] K. Heiskanen, Particle Classification, Chapman & Hall, London, 1993.
[32] T. Allen, Personal Communication, University of Bradford, Bradford, UK, 1980.
[33] T. Allen, Particle Size Measurement, 4th Edition, Chapman & Hall, London, 1990.
[34] K. Nageswararao, Reduced efficiency curves of industrial hydrocyclones — an analysis for plant practice, Min. Eng. 12 (1999) 517–544.
[35] D.F. Kelsall, A further study of the hydraulic cyclone, Chem. Eng. Sci. 2 (1953) 254–272.
[36] G.F. Brookes, N.J. Miles, J.S. Clayton, Hydrocyclone performance related to velocity parameters, in: Proceedings of the Second International Conference on Hydrocyclones, BHRA, Bath, UK, 1984, pp. 67–81.
[37] B.D. Rouse, J.S. Clayton, G.F. Brookes, Confirmation of modelling techniques for small diameter cyclones, in: Proceedings of the Third International Conference on Hydrocyclones, BHRA, Oxford, 1987, pp. 7–19.
[38] M. Frachon, J.J. Cilliers, A general model for hydrocyclones partition curves, Chem. Eng. J. 73 (1999) 53–59.
[39] E.J. Roldán-Villasana, R.A. Williams, T. Dyakowski, The origin of the fish-hook effect in hydrocyclone separator, Powder Technol. 77 (1993) 243–250.

[40] R. Del Villar, J.A. Finch, Modelling the cyclone performance with a size dependent entrainment factor, Min. Eng. 5 (1992) 661–669.
[41] D. Hodouin, S. Caron, J.J. Grand, Modelling and simulation of a hydrocyclone desliming unit, in: Proceedings of First World Congress on Particle Technology, Part IV, Nurnberg, 1986, pp. 507–522.
[42] J.A. Finch, Modelling a fish-hook in hydrocyclone selectivity curves, Powder Technol. 36 (1983) 127–129.
[43] M.B. Ray, A.C. Hoffmann, R.S. Postma, Performance of different analytical methods in evaluating grade efficiency of centrifugal separators, J. Aerosol. Sci. 31 (2000) 563–581.
[44] P. Rosin, E. Rammler, Discussion, J. Inst. Fuel 7 (1933) 109–112.
[45] D. Bradley, D.J. Pulling, Discussion, Trans. Inst. Chem. Engrs. 37 (1959) 44–45.
[46] D.F. Kelsall, Discussion, Trans. Inst. Chem. Engrs. 30 (1952) 104–108.